MAT 1341A –Draft Marking Scheme– Test 2, 2016

This is for version one only but can and should be used for each version.

3. Let $W = \{(x, y, z) \in \mathbf{R}^3 \mid x + y - z = 0\}$.

a) Explain *very briefly* why W is a subspace of $\mathbf{R}^3$. (*You will not need to use the Subspace Test.*)

Marks: 1

b) Find a spanning set for W.
[**1.5= 0.5 for knowing what 'spanning set' means + 1 all correct**]

c) Find a basis for W.

Marks: 2=1 (1 'showing' their vectors are l.i., + 1 all correct

d) Give a complete geometric description of W.

Marks: 1.5= .5 'plane' + .5 through (correct) 'p' + .5 with (correct) normal 'n'.

4. Let $\mathbf{M}_{22}$ denote the vector space of 2 by 2 matrices with real entries, and define

$$U = \left\{ \begin{bmatrix} a & b \\ -b & c \end{bmatrix} \in \mathbf{M}_{22} \mid a, b, c \in \mathbf{R} \right\}.$$

a) Either check that U is closed under addition, or express U in another form so you can simply state a theorem that guarantees that U is a subspace.

Marks: 1

b) Find a basis for U, and hence find $\dim U$.

Marks: 3= .5 matrices actually in U + 1 justification of l.i. + .5 any correct basis + .5 dim U consistent with their 'basis' +.5 correct dim U

c) Give a basis for U, different from the one you gave in (b).

Marks: 2 = .5 an answer consistent with (b) + .5 any correct answer +1 some decent attempt at justification

5. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example - with numbers, or functions, as is appropriate!

- If you say the statement is always true, you must give a clear explanation.

All :

Marks: [1.5= .5 (correct) + 1 justification (.5 is possible here)]

6. [Bonus] Suppose that u, v, w are non-zero vectors in $\mathbf{R}^{2016}$ such that $u \cdot v = u \cdot w = v \cdot w = 0$. Prove that $\{u, v, w\}$ is linearly independent.

(Your proof must work for *all* choices of non-zero vectors in $\mathbf{R}^{2016}$ such that $u \cdot v = u \cdot w = v \cdot w = 0$ — do not choose them yourself. Use the definition. No 'geometric' argument - e.g. "they are not co-planar" - will suffice, and in any case is meaningless to low-dimensional beings like your instructor and marker.)

Marks: [3 = 1 for some correct and useful idea + 1 for some good progress + 1 if all is correct and well-written. No ".5"s here.]